Algebra 1R – Mr. Bo
Unit 5 – Day 2 Notes

Name:_____
Date:_____

Graphs of Exponential Functions

Population Growth & Decline:

PROBLEM 1 Downtown and Uptown

At this moment, the population of Downtown is 20,000, and the population of Uptown is 6000. But over many years, people have been moving away from Downtown at a rate of 1.5% every year. At the same time, Uptown's population has been growing at a rate of 1.8% each year.

1. What is the initial population of Uptown? 6000 people.

2. Is the population of Uptown increasing or decreasing? At what rate?

Increasing $r = 0.018$

You can use the formula for compound interest to determine the function for Uptown's increasing population. Recall that the formula for compound interest is $P(t) = P(1 + r)^t$, where $P(t)$ represents the amount in the account after a certain amount of time in years, r is the interest rate written as a decimal, and t is the time in years.

3. Write a growth function, U(t), that represents Uptown's population growth as a function of time in years. What family of functions does U(t) belong? How do you know?

$U(t) = 6000(1.018)^t$ Exponential. the ind. variable is in the exponent.

4. Use the function to find the population of Uptown in 5 years.

$U(5) = 6000(1.018)^5$

$= 6,559$ people.

Now let's analyze the population decline of Downtown.

1. What is the initial population of Downtown? 20,000

2. Is the population of Downtown increasing or decreasing? At what rate?

Decreasing $r = -0.015$

3. Write a decline function, D(t), that represents Downtown's population decline as a function of time in years. What family of functions does D(t) belong? How do you know?

$D(t) = 20,000(1 - 0.015)^t$

$D(t) = 20,000(.985)^t$

Exponential because the t is in the exponent

4. Use the function to find the population of Downtown in 5 years.

$D(5) = 20,000(.985)^{(5)}$

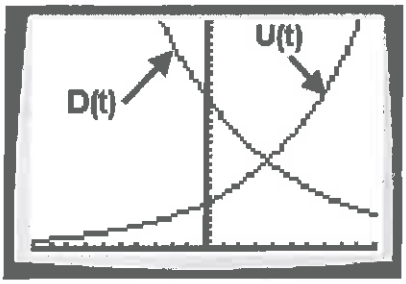
$= 18,544$ people.

PROBLEM 2 Graphing, Finally!

Let's examine the properties of the graphs of the functions for Downtown and Uptown. Here are the functions again:

Downtown: $D(t) = 20{,}000(1 - 0.015)^t$ Uptown: $U(t) = 6000(1 + 0.018)^t$

1. Use a graphing calculator to graph both functions using the window: [-100, 100]x[0, 30,000].



2. What is the y-intercept for each function? What do the y-intercepts represent in the context of the situations?

Downtown (0, 20,000)

uptown (0, 6,000) they represent the current populations.

3. Describe where the y-intercept of an exponential function can be found in the formula.

the y-int is the number being multiplied by $(1+r)^t$ part of the formula.

4. Use your calculator to answer each question and describe your strategy.

a. How long will it take for Uptown's population to double?

About 39 years. I graphed $y = 12{,}000$ on the calculator and used the intersect function to calculate the intersection.

b. How long will it take for the populations of Uptown and Downtown to be equal?

About 36.5 years. I used the intersect feature of the calculator to find where the graphs cross.

c. How long will it take for either population to reach zero? Describe any unusual results.

uptown will never reach zero because it started at 6,000 and continues to increase.

Downtown will not reach zero. the calculator produced an error when I graphed $y=0$ and tried to find the intersection.

Horizontal Asymptotes:

Each population function you graphed has a *horizontal asymptote*. A **horizontal asymptote** is a horizontal line that a function gets closer and closer to, but never intersects.

6. Write the equation for the horizontal asymptote of each population function.

Downtown: $y=0$ Uptown: $y=0$

7. Does the horizontal asymptote make sense in terms of this problem situation? Explain your reasoning.

no, since Downtown population continues to decline you would expect the population to reach zero at some point.

PROBLEM 3 The Multiple Representations of Exponentials

1. Based on the formula, will the graph increase or decrease? How do you know?

the graph should decrease because the base of $\frac{1}{2}$ is less than 1.

2. Based on the formula, what is the y-intercept of the graph? How do you know?

the y-int should be 1 since that is the # infront of the $(\frac{1}{2})^x$.

$1+r=\frac{1}{2}$

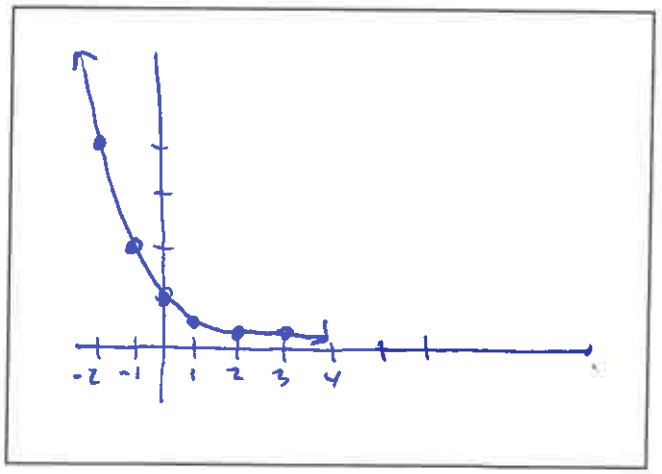
$r=-\frac{1}{2}$

the rate is negative.

3. Complete the table and sketch the graph.

b. $g(x)=\left(\frac{1}{2}\right)^x$

x	$g(x)$
−2	4
−1	2
0	1
1	1/2
2	1/4
3	1/8



x-intercept(s): none.

y-intercept: (0,1)

asymptote: $y=0$

domain: all Real #'s

range: $y>0$

interval(s) of increase/decrease:

decreasing for all real #'s.

1. Based on the formula, will the graph increase or decrease? How do you know?
Increase because the 2 is greater than 1.

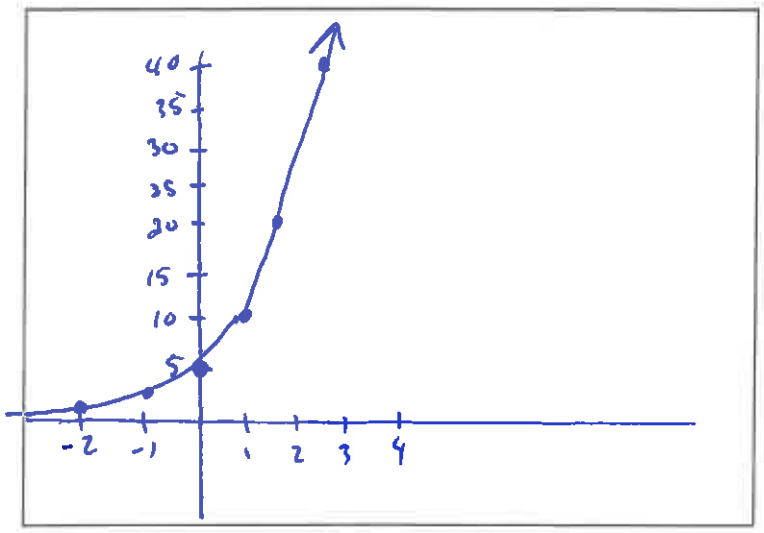
$1+r=2$
$r=1$ the rate is positive.

2. Based on the formula, what is the y-intercept of the graph? How do you know?
y-int. is (0,5) because 5 is the number in front of (2^x)

3. Complete the table and sketch the graph.

c. $k(x) = 5 \cdot 2^x$

x	k(x)
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40



x-intercept(s): none
y-intercept: (0,5)
asymptote: y=0
domain: all real #'s
range: y>0
interval(s) of increase/decrease:
Increasing for all real #'s.